2023

STATISTICS — GENERAL

Paper : GE/CC-3

(Introduction to Statistical Inference)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Give an example of a consistent estimator, which is biased for a population parameter.

(b) If F follows an F-distribution with 3, 2 degrees of freedom, then write down the probability density function of 1/F.

(c) What is critical region in the context of hypothesis testing?

(d) The p-value of a test statistic is obtained as 0.02. Will you accept the corresponding null hypothesis at 1% level of significance?

(e) What are the basic principles of Design of experiments?

(f) If X follows a normal distribution with mean μ and standard deviation σ, then what will be the distribution of $\frac{X-\mu}{\sigma}$ and $\left(\frac{X-\mu}{\sigma}\right)^2$?

(g) If a factor has 4 levels and each level has 5 observations, then find the degrees of freedom of error sum of squares in case of one-way ANOVA.

(h) State two advantages of using an RBD.

2. Answer ***any two*** questions :

(a) What is minimum variance unbiased estimator? If X, Y, and Z are independent unbiased estimators of θ and all have the same variance σ^2, which of the following estimators of θ will you prefer and why? 1+4

$$\frac{X+2Y+Z}{4}, \frac{2X+Y+2Z}{5}, \frac{X+Y+Z}{3}$$

(b) Distinguish between (i) null hypothesis and alternative hypothesis (ii) point estimation and interval estimation. 2½+2½

(c) Derive the partitioning of the total sum of squares of observations for a two-way ANOVA with 4 observations per cell into component sums of squares. Write down the corresponding component mean sums of squares. 3+2

Please Turn Over

3. Answer *any three* questions :

(a) If a random sample of size n is drawn from $N(\mu, \sigma^2)$ distribution, show that the sample mean and the sample variance (having divisor $n-1$) are independently distributed. Also find their distributions. 7+3

(b) Describe the test procedure for testing the null hypothesis $H_0 : \sigma^2 = \sigma_0^2$ on the basis of a random sample of size n drawn from $N(\mu, \sigma^2)$, where μ is unknown. Also find the $100(1-\alpha)\%$ confidence interval for σ^2. 6+4

(c) What do you mean by Analysis of Variance (ANOVA)? Describe the ANOVA technique for a two-way classified data with one observation per cell under the fixed effects model. 2+8

(d) What is CRD? Explain clearly the layout and analysis of CRD. Write two disadvantages of CRD. 2+6+2

(e) (i) If X, Y and Z are three statistics with expectations $E(X) = 3\theta_1 + 2\theta_2 + \theta_3$, $E(Y) = 2\theta_1 + 3\theta_2 + \theta_3$, $E(Z) = \theta_1 + \theta_2 + \theta_3$, find unbiased estimators of θ_1, θ_2, θ_3 and $\theta_1 - 2\theta_2 + \theta_3$.

(ii) What is the relation between F and χ^2 distributions? 8+2
